NAME (please print legibly): ________________________________________________________
Your University ID Number: _____________________________________________________

Please complete all questions in the space provided. You may use the backs of the pages for extra
space, or ask me for more paper if needed. This exam will be graded on:

• Correctness of computations.

• Clarity of explanation of procedure.

• Correctness of procedure.

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for
full credit. Please feel free to write as much as you like. Work carefully, and try to complete the
problems you find easier before going back to the harder ones. Good luck!

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<th>QUESTION</th>
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<td>TOTAL</td>
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</table>
1. (15 points) A fair die is rolled, producing an integer $n$ between 1 and 6. What is the sample space $S$? (5pts)

ANSWER: ____________________________

Give an example of an event $A$ containing 3 outcomes and an event containing 2 outcomes. (5pts)

ANSWER: ____________________________

Define disjoint events and state whether event $A$ and event $B$ are disjoint. (5pts)

ANSWER: ____________________________

2
2. (10 points) State the inclusion-exclusion rule for three events, $A$, $B$, and $C$ (5pts):

Suppose we know that $P(A \cup B \cup C) = 0.9$, $P(B) = 0.6$, $P(C) = 0.3$, $P(A \cap B) = 0.2$, $P(A \cap C) = 0.1$, $P(B \cap C) = 0.1$, and $P(A \cap B \cap C) = 0$. Find $P(A)$. (5pts)

ANSWER: _______________________

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ANSWER: _______________________

3
3. **(15 points)** A random integer \( n \) is chosen between 2 and 20 (with equal probabilities). Compute the probability of the following events (5pts):

\[
A = \{ \text{n is odd} \}, \quad B = \{ \text{n is prime} \}, \quad C = \{ \text{n is even} \}
\]

**ANSWER:**

State the definition of independent events (5pts):

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**ANSWER:**
(Question 3, continued) Consider the pairs of events \( A, B, A, C \) and \( B, C \) from the previous page. Which are dependent and which are independent? Justify your answer using the definition of independence for full credit. (5pts)

**Answer:** __________________________
4. (10 points) A bag contains 4 marbles, 2 of which are red and 2 are black. We draw the marbles from the bag (randomly) one after the other, \textbf{without replacing them}. Consider the events:

\[ A = \{ \text{the first marble is red} \}, \quad B = \{ \text{the second marble is red} \} \]

Compute \( P(A) \) and \( P(B) \). (5pts)

\textbf{ANSWER:} ______________

Are \( A \) and \( B \) independent events? Justify your answer using the definition of independence for full credit. (5pts)

\textbf{ANSWER:} ______________
5. (20 points) State the definition of \textit{mutual} independence for 3 events \(A, B,\) and \(C\) (5pts).

\textbf{ANSWER:} ____________________________

Two (fair) dice are rolled, producing a pair of integers \(x_1\) and \(x_2\) between 1 and 6. Consider the three events

\[ A = \{(x_1, x_2)|x_1 + x_2 = 7\} \quad B = \{(x_1, x_2)|x_1 = 3\} \quad C = \{(x_1, x_2)|x_2 = 4\} \]

Find \(P(A), P(B),\) and \(P(C).\) Justify your answer for full credit. (5pts)

\textbf{ANSWER:} ____________________________
(Continued). Are the pairs of events $A, B$, $A, C$, and $B, C$ independent or dependent? Justify your answer using the definition of independence for full credit. (5pts)

ANSWER: ________________________

Are all three events $A$, $B$, and $C$ jointly (or mutually) independent? Justify your answer for full credit using the definition of mutual independence you gave above. (5pts)

ANSWER: ________________________