MATH 2250

Final Exam

May 2, 2012

NAME (please print legibly): _____

Your University ID Number: _____

Problems that ask for an explanation will be graded on:

- Correctness of computations.
- Clarity of explanation of procedure.
- Correctness of procedure.

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for full credit. Please feel free to write as much as you like. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

Remember that you are **required** to have a non-graphing calculator to complete the exam. Remember also that smartphone (or computer, or other device) use is **prohibited** on this exam, regardless of what you use it for.

QUESTION	VALUE	SCORE
1	35	
2	10	
3	10	
4	20	
5	10	
6	15	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
TOTAL	160	

1. (35 points) Differentiate the following functions. No explanation is required. Please box your answers. (Grading: 5 pts each, no partial credit)

$y = x\cos(x)$

$y = (3x+1)^{99}$

$y = 2x^2(3x+1)$

$y = \frac{x+2}{x+4}$

$y = \sqrt{3x^2+1}$

$y = \frac{1}{\sqrt{2x+1}}$

$y = \csc(\sin(2x))$

2. (10 points) If the function $f(x)$ is given by

$$f(x) = \tan x$$

Write the first three terms of the Taylor series for $f(x)$ at $x = 0$ as a polynomial in Δx.

$f(x + \Delta x) =$

ANSWER: _____

3. (10 points) Find the tangent line to

$$f(x) = \arctan x + 2$$

at $x = 4$. Please explain your work.

ANSWER: _____

4. (20 points) The acceleration of gravity on Jupiter's ice moon Europa is $g = -1.314\ (\mathrm{m/s^2})$. An astronaut on the first human mission to Europa steps off a 10 meter cliff. How long does it take her to fall to the surface? What is her speed when she impacts the ice at the base of the cliff? Justify your best guess: Does she survive? (It may help to convert m/s to the more familiar units of mph. Recall that there are about 1610 meters in 1 mile.) Please explain your work.

ANSWER: _____

5. (10 points) Heron's formula for the area of a triangle with sides x, y, and z is given by

$$S(x, y, z) = \frac{1}{4}\sqrt{(x + y + z)(x + y - z)(x - y + z)(-x + y + z)}$$

Compute the area $S(2, 3, 5)$ of a triangle with sides $x = 2$, $y = 3$ and $z = 5$. Differentiate $A(x) = S(x, 3, 5)$ with respect to x at $x = 2$ and compute the derivative $A'(2)$.

ANSWER: _____

6. (15 points) Use Newton's method to solve the equation

$$x^4 - x = 7$$

Grading: 5 points for knowing Newton's method + 2 points per correct digit to the right of the decimal place (max of 15 pts total). BOX YOUR FINAL ANSWER.

ANSWER: _____

7. (10 points) Find the maximum and minimum values of the function

$$f(x) = \frac{1}{3}x^3 - \frac{6}{2}x^2 + 8x - 11.$$

on the interval $[0, 3]$ using calculus. Please explain your work.

ANSWER: _____

8. (10 points) Osbourne discovered in 1909 that the radius r of a spherical rubber balloon is related to the pressure in the balloon by the equation

$$r\left(\frac{2K}{r_0} - p\right) = 2K$$

where K is a constant related to the elasticity of the rubber and r_0 is the initial (uninflated) radius of the balloon. **Use implicit differentiation** to find $\frac{d}{dr}p$ for a balloon with $K = 2$ and $r_0 = 1$ (m) at $r = 2$ (m). Please explain your work (but you don't need to write out differentiation rules).

ANSWER: _____

9. (10 points) (Error Problem) The field strength E of a radio signal at a receiving station is given as a function of transmitter power P (watts) and distance d (m) between transmitter and receiver by

$$E = \frac{\sqrt{30P}}{d}$$

Suppose that a cell phone transmitting at 4 ± 0.1 watts is 5 km $(5000$ m) from the nearest cell tower. Estimate the resulting range of field strengths at the tower using our error formula.

ANSWER: _____

10. (10 points) Evaluate the indefinite integral

$$\int \sec x \tan x \, dx,$$

then differentiate your answer to be sure that it is correct.

ANSWER: _____

11. (10 points) Evaluate the integral

$$\int_2^4 \frac{1}{x^2 - 4x + 100}\, dx,$$

Remember to explain any u-substitution that you use and to watch the limits of integration.

ANSWER: _____

12. (10 points) Evaluate the integral

$$\int_1^3 \frac{1}{x^2 - 4x + 3}\, dx.$$

Remember to explain any u-substitution that you use and to watch the limits of integration.

ANSWER: _____