Math 2500 - Lecture 1.

About me, syllabus, name game/pics/anki  25 min
Why multivariable calculus?  25 min

Derivatives. Maximize $f(\theta) = \cos \theta \sin \theta$. 
(car launch problem)
Take derivative.
Find critical point.
What this means for $f(\theta, y) = \cos \theta^2 y$.

Integrals. Integrate $\int_{-1}^{1} 1 - x^2 \, dx$. 
(area under line)
Integrate $\iint \left[ 1 - (x^2 + y^2) \right] \, dx \, dy$
(volume under surface)

Later in course we'll have some entirely new ideas.
Math 2500 - Lecture 2 - Vectors.

Definition. A point in $\mathbb{R}^n$ is an ordered list of $n$ numbers $(x_1, \ldots, x_n)$.

Examples. $\mathbb{R}^2, \mathbb{R}^3, \mathbb{R}^n$

Definition. A vector $\vec{v}$ in $\mathbb{R}^n$ is a directed line segment joining two points in $\mathbb{R}^n$.

Convention. If the first point is not given, we assume it's the origin.

Examples. $\vec{v} = (2, 4) (3, 5), \vec{v} = (7, 8)\vec{i} + (7, 8)\vec{j}$.

Definition. The length (or magnitude) of $(x_1, \ldots, x_n) = \vec{x}$ is $\sqrt{x_1^2 + \ldots + x_n^2}$, denoted $||\vec{x}||$.

Definition. $\vec{u} + \vec{v} = (u_1 + v_1, \ldots, u_n + v_n)$, $k\vec{u} = (ku_1, \ldots, ku_n)$.

Note: We call a number $k$ a scalar to avoid confusion.

Example. Addition, with pictures, scalar mult, with pics.

Example. Distance formula, midpoint, medians of triangle.

Exercises. $\vec{x} + \vec{y} = \vec{y} + \vec{x}$. $(\vec{x} + \vec{y}) + \vec{z} = \vec{x} + (\vec{y} + \vec{z})$. $c(\vec{x} + \vec{y}) = c\vec{x} + c\vec{y}$. 
Math 2500 – Lecture 3 - Dot product, cross product

By similar triangles \( \frac{X_2}{X_1} = \frac{-Y_1}{Y_2} \), or \( x_1y_2 + x_2y_1 = 0 \).

Definition. The dot product \( \mathbf{u} \cdot \mathbf{v} = u_1v_1 + \ldots + u_nv_n \).

Theorem. The angle \( \theta \) between \( \mathbf{u} \) and \( \mathbf{v} \) is given by \( \cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}|| \ ||\mathbf{v}||} \).

Proof. Law of cosines. \( c^2 = a^2 + b^2 - 2ab \cos \theta \).

Corollary. \( \mathbf{u} \perp \mathbf{v} \iff \mathbf{u} \cdot \mathbf{v} = 0 \).

Examples. \((3,2), (4,6), (3,2), (7,13)\)

Exercises. \( \mathbf{u} \cdot \mathbf{v} = \vec{u} \cdot \vec{v} \), \((c\vec{u}) \cdot \vec{v} = c \vec{u} \cdot \vec{v} = c(\vec{u} \cdot \vec{v}) \), \( \mathbf{u} \cdot (\mathbf{u} + \mathbf{v}) = \mathbf{u} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v} \\ \mathbf{u} \cdot \mathbf{v} = ||\mathbf{u}||^2 \), \( \theta = \frac{\pi}{2} \).

Proposition. \( \text{proj}_\mathbf{u} \mathbf{v} = \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}||} \mathbf{v} \).

Examples. \( ||\mathbf{x} \cdot \mathbf{y}|| \leq ||\mathbf{x}|| \ ||\mathbf{y}|| \) (Cauchy-Schwarz)

Definition. The cross product of two vectors \( \mathbf{u}, \mathbf{v} \in \mathbb{R}^3 \) is \( \mathbf{u} \times \mathbf{v} = (x_2y_3 - x_3y_2, x_3y_1 - x_1y_3, x_1y_2 - x_2y_1) \).

Right-hand rule. \( \mathbf{u} \times \mathbf{v} = (||\mathbf{u}|| \ ||\mathbf{v}|| \sin \theta) \hat{n} \).

Corollary. \( \mathbf{u} \) and \( \mathbf{v} \) are parallel \( \iff \mathbf{u} \times \mathbf{v} = \mathbf{0} \).

Exercises. \((r\mathbf{u}) \times (s\mathbf{v}) = rs(\mathbf{u} \times \mathbf{v}) \), \( \mathbf{u} \times (\mathbf{u} + \mathbf{v}) = \mathbf{u} \times \mathbf{u} + \mathbf{u} \times \mathbf{v} \).

\( \mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u} \), \( \mathbf{0} \times \mathbf{u} = \mathbf{0} \).

Examples. \((1,3,1) \times (2,4,5), (1,3,1) \times (2,6,2)\)

Proposition. \( \mathbf{u} \times \mathbf{v} \) is the area of a parallelogram.

Definition. \((\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} \) is called the triple scalar product.

Proposition. \((\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \text{volume of box} \).