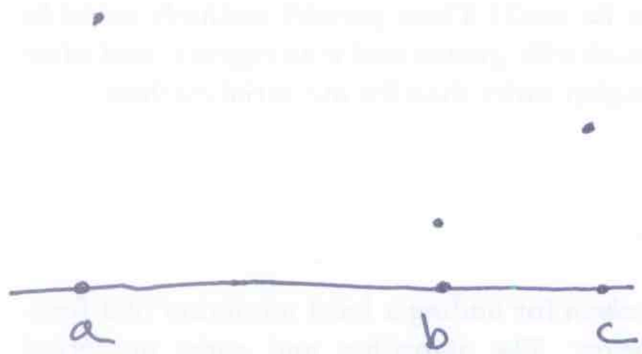
Minimization with derivatives

Suppose we have not only $f(x)$ but $f'(x)$ available to us and we want to minimize $f(x)$ on $[a,b]$.

Remark. If you are approximating $f'(x)$ using $f(x)$ evaluations using one of the methods that we've covered before, you DON'T have $f'(x)$ available (roundoff error will kill you, and you'll be better off with a straight Brent's method code - see Mathematica demo).

So suppose we have a triple



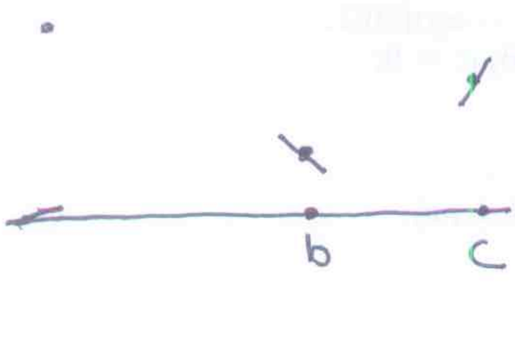
and we know there's a min in (a,c) because $f(b) < f(a), f(c)$.

Observe that the sign of $f'(b)$ can be used to predict whether (a,b) or (b,c) contains the min.

Further if we know (say)

$f(b), f'(b) < 0$

$f(c), f'(c) > 0$



we can guess the root of f'
between b and c using the
secant line method.

(The secant method converges
superlinearly, so we get superlinear
convergence near the min.)

If our prediction for the root of f'
is outside (b,c), or too close to
the endpoints to evaluate stably,
we will just bisect (b,c).

This gives you the derivative
form of Brent's method (due to Brent).

(mathematica demo)

You can read about this in the excerpt from "Numerical Recipes" (Section 10.3) posted on the course page.