Good before bad.

Suppose we have a sample space S with a partition into 3 events, G, B, N, so

$$P(G) = p, \quad P(B) = q, \quad P(N) = r$$

($p + q + r = 1$, since this is a partition)

We repeat independent trials. What is the probability that we get an event in G before we get an event in B?

Let A_n be the event

$$\{(x_1, x_2, \dots) \mid x_j \in N \text{ for } j < n,\ x_n \in G\}$$

that is, all trials are neutral

until the n-th trial, which is good.

Note. The A_n are disjoint events.

$$P(A_n) = P(x_1 \in N \cap x_2 \in N \cap \ldots \cap x_{n-1} \in N \cap x_n \in G)$$

$$= P(N) \times P(N) \times \ldots \times P(N) \times P(G)$$

$$= P(N)^{n-1} P(G)$$

$$= r^{n-1} p.$$

So P(good before bad) is

$$P\left(\bigcup_{j=1}^{\infty} A_j\right) = \sum_{j=1}^{\infty} r^{n-1} p$$

But this is a geometric series!

Recall

$$\sum_{j=1}^{\infty} r^{n-1} p = \frac{p}{1-r} \quad \text{as long as } r<1$$

$$= \frac{p}{p+q}$$

Example 1. Double dice.

A player rolls a standard d6. On that die, the numbers on opposite sides add to 7, and we call $7-n$ the <u>opposite</u> roll.

Rules. Roll and note the outcome. X_1

Roll until you repeat X_1 (you win) or get $7-X_1$ (you lose).

What are your odds of winning? Do they depend on X_1?

~~Variant. Pick a number between 1 and 6, to be X_1.~~