

Math 8230: “Grassmannians and Stiefel Manifolds” Syllabus

1. COURSE INFORMATION

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Boyd 326

1:25-2:15 MWF

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2. COURSE SCHEDULE

- (1) Basic Definitions
 - (a) Classical Matrix Groups (and their algebras)
 - (i) Gallier, “Manifolds, Lie Groups, Lie Algebras, Riemannian Manifolds, with Applications to Computer Vision and Robotics”, Notes 1.
 - (b) Grassmannians and Stiefel Manifolds as Quotients of Classical Groups
 - (c) The Polar Decomposition and Full-Rank Matrices
 - (i) Gallier, “Manifolds, Lie Groups, Lie Algebras, Riemannian Manifolds, with Applications to Computer Vision and Robotics”, Notes 2.
- (2) The Topology of Stiefel Manifolds
 - (a) Fiber Bundles, Homotopy Groups, and Long Exact Sequence
 - (i) Hatcher, 375-384.
 - (b) Cell Structure and Schubert Calculus, Homology Groups
 - (i) Ledoux and Malham, “Introductory Schubert Calculus”.
 - (ii) Hatcher, 292-302.
 - (iii) Jianwei, “The Geometry and Topology on Grassmann Manifolds”.
- (3) The Geometry of Stiefel Manifolds
 - (a) Riemannian Structure, Distances, and Angles.
 - (i) Edelman, Arias, Smith, “Geometry of Algorithms with Orthogonality Constraints”.
 - (ii) Paige, Wei, “History and Generality of the CS Decomposition”.
 - (b) Geodesics and Projections.
 - (i) Younes, Michor, Shah, Mumford. “A Metric on Shape Space with Explicit Geodesics”.
- (4) Applications of Grassmann and Stiefel Manifolds.
 - (a) Polygon Spaces
 - (i) Knutson and Hausmann, “Polygon Spaces and Grassmannians”.
 - (b) Frame Spaces
 - (c) Probability Distributions
 - (i) Chikuse, “Statistics on Special Manifolds”.

3. PREREQUISITES

The general prerequisite for the course is a certain level of “mathematical maturity”, usually equivalent to completing the first-year graduate courses, including the algebraic topology sequence. While background in algebraic geometry, Riemannian geometry, Lie groups, and numerical analysis would certainly be helpful at various points in the course, it is expected that essentially every student will be unfamiliar with the basic terms at *some* point in the semester. For this reason, we’ll try to give mini-introductions when we switch fields.

4. COURSE GOALS

The goal of the course is for the students to get an overview of the many roles played by Stiefel and Grassmann manifolds throughout pure and applied mathematics. The hope is that by understanding the various tools and techniques developed in other areas, you can draw some interesting new conclusions in your “home” fields of mathematics.

5. DISCLAIMER

“The syllabus is a general course plan, but deviations may become necessary over the course of the semester.” Among other things, we may change the dates of the exams if the class runs slower or faster than expected. In this case, we’ll announce the change several times in class: if you miss the exam I won’t be able to help you.

6. PRINCIPAL COURSE ASSIGNMENTS

The principal assignments in this course are a set of irregular homework assignments.

7. GRADING AND WP/WF POLICY

Grades will be determined from class participation (30%) and homework (70%). It is required to set a formal policy for grades of “WP” and “WF”. Here is mine: every student who withdraws will get a grade of “WP” if they have attended class.

8. ATTENDANCE POLICY

None.

9. ACADEMIC HONESTY

As a University of Georgia student, you have agreed to abide by the University’s academic honesty policy, “A Culture of Honesty,” and the Student Honor Code. All academic work must meet the standards described in *A Culture of Honesty* found at: www.uga.edu/honesty. Lack of knowledge of the academic honesty policy is not a reasonable explanation for a violation. Questions related to course assignments and the academic honesty policy should be directed to the instructor.

You are permitted to work together on homework assignments. It is even reasonable to search the internet for related problems. The key thing is that you write up the assignments by yourself, using your own words. You won’t learn anything otherwise!

10. REQUIRED COURSE MATERIAL

There is no book: we will work from papers posted on the course website.