MATH 2550

Final Exam

December 8, 2006

NAME (please print legibly): ____

Please complete all 10 questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed. Work carefully, and try to complete the problems you find easier before going back to the harder ones.

Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
TOTAL	120	

1. (10 points) Two curves $f(t)$ and $g(t)$ are given by

$$f(t) = \frac{\sqrt{2}}{2}(\cos t, \sin t, t) \quad \text{and} \quad g(t) = \left(\frac{1}{2}t^2, \frac{2\sqrt{2}}{3}t^{3/2}, t\right).$$

(Sorry about the weird-looking coefficients!) where t ranges from 0 to 10. Which curve is longer?

ANSWER: ____________________________

2. (10 points) A function $f(x, y)$ has the gradient vector $\nabla f = (3, 4)$ at the point $(0, 0)$.

What is the directional derivative of f in the direction $(1, 4)$?

ANSWER: ____________________

Find a direction $\vec{v}$ in which the directional derivative of f is equal to zero.

ANSWER: ____________________

Find the **unit** vector $\vec{v}$ in which the directional derivative of f is **least**.

ANSWER: ____________________

3. (10 points) Find the partial $\frac{\partial^2}{\partial x \partial y} xy \sin(x+y)$.

ANSWER: ____________________

The mixed partial $\frac{\partial^5}{\partial x^2 \partial y^3} = \frac{\partial^5}{\partial y^3 \partial x^2}$ is zero for the following functions. Is is easier to show this by differentiating first with respect to x or differentiating first with respect to y?

1. xe^{y^2} ANSWER: ____________________

2. $x^2 + 5xy + \sin x + 7e^x$ ANSWER: ____________________

3. $y^2 x^4 e^x + 2$ ANSWER: ____________________

4. (10 points) Find three real numbers a, b, and c with sum equal to 9 so that the sum of their squares is as small as possible.

ANSWER: __________________________

5. (10 points) The vector field V is given by

$$V = \left(x\cos(x^2+y^2+z^2), y\cos(x^2+y^2+z^2), z\cos(x^2+y^2+z^2)\right).$$

Use any method to compute the work integral

$$\int V \cdot \mathrm{d}s$$

over the curve $r(t) = (t^2, \sqrt{2}t, 1)$ from $t = 0$ to $t = 5$.

ANSWER: ______________________________

6. (10 points) Calculate the counterclockwise circulation integral

$$\int (y^2, x^2) \cdot \mathrm{d}s$$

around the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$.

ANSWER: ____________________________

7. (10 points) One of the three vector fields below is the gradient of a function. Which one? And what function?

1. $V = (xy, xy)$
2. $W = (y \sin x, y \cos x)$
3. $X = (y, x)$

ANSWER: ______________________

8. (10 points) A function f obeys the equation

$$\frac{\partial^2}{\partial x^2} f + \frac{\partial^2}{\partial y^2} f = 0.$$

on the interior of the circle of radius 1. Find the counterclockwise circulation integral around the circle of radius 1

$$\int \left(\frac{\partial f}{\partial y}, -\frac{\partial f}{\partial x} \right) \cdot ds$$

ANSWER: ____________________________

9. (10 points) Find the area of the portion of the helical surface

$$r(u, v) = (u \cos v, u \sin v, u)$$

with u between 0 and 1 and v between 0 and 2π.

ANSWER: ____________________

10. (10 points) Suppose that R is a region in space with outward pointing normal $\vec{n}$. Explain why the **volume** of R can be computed with the **surface** integral

$$\text{Volume}(R) = \frac{1}{3} \iint_{\partial R} (x, y, z) \cdot n \, dA.$$

Then use this formula to derive an equation for the volume of the sphere of radius r.

ANSWER: ____________________________

11. (10 points) **[Challenge Problem]** The polar moment of a region R in the plane is the integral

$$\int_R x^2 + y^2 dA$$

A triangle in the plane has vertices $(0,0)$, $(a,0)$ and $(a,1/a)$. Find the value of a which minimizes the polar moment.

ANSWER: ____________________________

12. (10 points) [Challenge Problem] Compute the outward flux of the vector field V over the portion of the sphere of radius 2 in the first octant where

$$V = \left(x^2, -2xy, 3xz\right)$$

and the first octant is the region of space with x, y, and z all ≥ 0.

ANSWER: ____________________________