Problem Session 1.

Use the arithmetic-geometric mean inequality to show that among all triangles of perimeter $p$, the equilateral one has the most area.

Show that for positive $x, y, \alpha, \beta$

$$x^\alpha y^\beta \leq \frac{\alpha}{\alpha + \beta} x^{\alpha + \beta} + \frac{\beta}{\alpha + \beta} y^{\alpha + \beta}$$

and hence $x^{2004} y + x y^{2004} \leq x^{2005} + y^{2005}$.

(Harder) Prove $a + b + c \leq \frac{a^3}{bc} + \frac{b^3}{ac} + \frac{c^3}{ab}$. 