1. a. \( \vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix} \). Compute \((\vec{x}+\vec{y}) \cdot (\vec{x}-\vec{y})\) using any method you like.

b. Suppose \( \vec{x}, \vec{y} \in \mathbb{R}^n \) each contain the numbers \( 1, \ldots, n \) as coordinates, in some order (as in part a). Compute \((\vec{x}+\vec{y}) \cdot (\vec{x}-\vec{y})\).

2. The linear transformation \( T_\theta \) is defined by rotating the plane counterclockwise by \( \theta \) radians. Find the standard matrix for \( T_\theta \).

3. The space of \( n \times n \) matrices is the same as \( \mathbb{R}^{n^2} \) (just write out all the entries as a long vector). Prove or disprove: The set of skew-symmetric matrices is a subspace of \( \mathbb{R}^{n^2} \).

Note: Problems with a star are a little harder!
4. Find a matrix $A$ for which $A^{10} = I$, but $A^9 \neq I$. (Hint: Let $A$ be a $2 \times 2$ matrix, so it represents a linear transformation of the plane).

5. Suppose that $A^3 = 0$. Prove that $I + A + A^2$ is an invertible matrix. (Find an explicit formula for $(I + A + A^2)^{-1}$).

6. Define:
   a) upper-triangular
   b) main diagonal
   c) linear map
   d) cross product

Bonus question: Given a square ($mn$) matrix $A$, suppose $A \vec{x} \cdot A \vec{y} = \vec{x} \cdot \vec{y}$ for all $\vec{x}, \vec{y} \in \mathbb{R}$. Prove that $A^T = A^{-1}$.

Note: Problems with a star are a little harder!