Math 4510/6510 Homework #2

1. Find coefficients $a$, $b$, $c$, $d$, and $e$ so that

$$x(t+h) = a x(t) + b x(t-h) + h \left[ c x'(t+h) + d x''(t) + e x'''(t-h) \right]$$

holds for polynomials $x(t)$ of as high a degree as possible.

2. Why do the coefficients in the Adams-Bashforth formulas add up to 1?

3. Determine the numerical value of

$$2\pi \int_4^5 \frac{e^x}{x} \, dx$$

in three ways: numerical integration, solving the ODE numerically, and integrating symbolically and evaluating the exact formula. Is it better to numerically integrate? Or numerically solve the ODE?

4. The second-order Adams-Bashforth-Moulton method is given by

$$\tilde{x}(t+h) = x(t) + \frac{h}{2} \left[ 3f(t, x(t)) - f(t-h, x(t-h)) \right]$$

$$x(t+h) = x(t) + \frac{h}{2} \left[ f(t+h, \tilde{x}(t+h)) + f(t, x(t)) \right]$$

The approximate single-step error is

$$\epsilon \simeq \frac{1}{6} |x(t+h) - \tilde{x}(t+h)|$$

Using Mathematica, write and test an adaptive procedure for solving an ODE (you can pick the ODE) which uses this method and uses $\epsilon$ to monitor convergence and adjust step size accordingly.

5. Explain how to solve the system

$$x'_1(t) = x_1(t)e^t + \sin t - t^2$$

$$x'_2(t) = \left[ x_2(t) \right]^2 - e^t + x_2(t)$$

$$x_1(1) = 2, \quad x_2(1) = 4$$

using only a solver for equations in the form $x'(t) = f(t, x(t))$ for a single function $x(t)$.

6. Convert the differential equation

$$x'''(t) = t + x + 2x' + 3x''$$

$$x(1) = 3$$

$$x'(1) = -7$$

$$x''(1) = 4$$

into a first-order system of ODE.

7. Solve Airy’s equation:

$$x'' = t \, x, \quad x(0) = 0.355028053887817, \quad x'(0) = -0.258819403792807$$

using your own code in Mathematica. You can check your code with $x(4.5) = 0.0003302503$, which is correct.