sqr

Purpose

Sparse QR factorization.

Synopsis

[R,p]=sqr(A)
[R,p,c]=sqr(A,b)

Description

`[R,p]=sqr(A)` computes the upper triangular n-by-n matrix R in the QR factorization of the sparse m-by-n, $m \geq n$, matrix $A(r,p)$,

$$Q^T A(r,p) = \begin{pmatrix} R \\ 0 \end{pmatrix}.$$

Here, p is a postordered column ordering and r the row ordering used. Since R is invariant under row orderings of A, r is not made available for the user.

`[R,p,c]=sqr(A,b)` does also compute the n first components of $Q^T b(r)$. This vector/matrix is often used in the solution of sparse linear least squares problems.

Example

Sparse linear least squares problems, $\min_x ||Ax - b||_2$, are easily solved by the use of `sqr`. A fill-in minimizing column ordering q is first computed. Then the resulting matrix is factorized and the least squares solution computed from the triangular system $Rx = Q^T b$. Since `sqr` computes the QR factorization of a column permutation of A, the inverse ordering must be applied to the computed solution.

```
q=colmmd(A);             % Minimum degree ordering of A.
[R,p,c]=sqr(A(:,q),b);   % Sparse QR-factorization.
Pc=q(p);                 % Resulting column permutation.
x=R\c;                   % Least squares solution
x(Pc)=x;                 % Permute the solution.
```

Algorithm

The upper triangular matrix R is computed by a multifrontal scheme described by Matstoms [1]. For each node in the elimination tree of $A^T A$, a dense frontal matrix is formed and factorized. The first rows of the factor define one or more rows of R, and the remaining part forms an update matrix to be included in the frontal matrix of the father node.

References

[1] P. Matstoms, *Sparse QR factorization in MATLAB*, ACM Trans. Math Software, ? (1993), To appear.